A microscopic estimate of the nuclear matter compressibility and symmetry energy in relativistic mean-field models

D. Vretenar and T. Nikšić

Physics Department, Faculty of Science, University of Zagreb, Croatia, and Physik-Department der Technischen Universität München, D-85748 Garching, Germany

P. Ring

Physik-Department der Technischen Universität München, D-85748 Garching, Germany

(Dated: June 6, 2005)

Abstract

The relativistic mean-field plus RPA calculations, based on effective Lagrangians with density-dependent meson-nucleon vertex functions, are employed in a microscopic analysis of the nuclear matter compressibility and symmetry energy. We compute the isoscalar monopole and the isovector dipole response of ^{208}Pb, as well as the differences between the neutron and proton radii for ^{208}Pb and several Sn isotopes. The comparison of the calculated excitation energies with the experimental data on the giant monopole resonance in ^{208}Pb, restricts the nuclear matter compression modulus of structure models based on the relativistic mean-field approximation to $K_{\rm nm} \approx 250-270$ MeV. The isovector giant dipole resonance in ^{208}Pb, and the available data on differences between neutron and proton radii, limit the range of the nuclear matter symmetry energy at saturation (volume asymmetry) to 32 MeV $\leq a_4 \leq$ 36 MeV.

PACS numbers: 21.60.Ev, 21.60.Jz, 21.65.+f, 24.30.Cz

I. INTRODUCTION

Basic properties of nuclear ground states, excitation energies of giant monopole resonances, the structure of neutron stars, the dynamics of heavy-ion collisions and of supernovae explosions, depend on the nuclear matter compressibility. The nuclear matter compression modulus $K_{\rm nm}$ is defined as

$$K_{\rm nm} = k_f^2 \frac{d^2 E/A}{dk_f^2}\Big|_{k_{f_0}} , \tag{1}$$

where E/A is the binding energy per nucleon, k_f is the Fermi momentum, and k_{f_0} is the equilibrium Fermi momentum. The value of $K_{\rm nm}$ cannot be measured directly. In principle it can be extracted from the experimental energies of isoscalar monopole vibrations (giant monopole resonances GMR) in nuclei. Semi-empirical macroscopic leptodermous expansions, as well as microscopic calculations, have been employed in the analysis of available data on isoscalar GMR. Although macroscopic expansions, analogous to the liquid drop mass formula, in principle provide "model independent" estimates of $K_{\rm nm}$, in reality they do not constrain its value to better than 50%.

A more reliable approach to the determination of $K_{\rm nm}$ is based on microscopic calculations of GMR excitation energies. Self-consistent mean-field calculations of nuclear ground state properties are performed by using effective interactions with different values of $K_{\rm nm}$. Interactions that differ in their prediction of the nuclear matter compressibility, but otherwise reproduce experimental data on ground-state properties reasonably well, are then used to calculate GMR in the random phase approximation or the time-dependent framework. A fully self-consistent calculation of both ground-state properties and GMR excitation energies restricts the range of possible values for $K_{\rm nm}$. The correct value of $K_{\rm nm}$ should then be given by that interaction which reproduces the excitation energies of GMR in finite nuclei. It has been pointed out, however, that, since $K_{\rm nm}$ determines bulk properties of nuclei and, on the other hand, the GMR excitation energies depend also on the surface compressibility, the determination of the nuclear matter compressibility relies more on a good measurement and microscopic calculations of GMR in a single heavy nucleus such as ^{208}Pb, rather than on the systematics over the whole periodic table [1, 2]. It has also been emphasized that the determination of a static quantity, $K_{\rm nm}$, from dynamical properties, i.e. from GMR energies, is potentially ambiguous. Various dynamical effects, as for instance the coupling of single particle and collective degrees of freedom, could modify, though not much, the deduced value of the nuclear matter compression modulus. On the other hand, it has been shown that $K_{\rm nm}$ cannot be extracted from static properties, i.e. masses and charge distributions, alone [3].

In this work we address a different source of ambiguity, which has become apparent only recently. Modern non-relativistic Hartree-Fock plus random phase approximation (RPA) calculations, using both Skyrme and Gogny effective interactions, indicate that the value of $K_{\rm nm}$ should be in the range 210-220 MeV [1, 2]. In Ref. [3] it has been shown that even generalized Skyrme forces, with both density- and momentum-dependent terms, can only reproduce the measured breathing mode energies for values of $K_{\rm nm}$ in the interval 215 ± 15 MeV. A comparison of the most recent data on the $E0$ strength distributions in ^{90}Zr, ^{116}Sn, ^{114}Sm and ^{208}Pb, with microscopic calculations based on Gogny effective interactions by Blaizot *et al.* [1], has put the value of $K_{\rm nm}$ at 231 ± 5 MeV [4]. In relativistic mean-field (RMF) models on the other hand, results of both time-dependent and RPA calculations suggest that empirical GMR energies are best reproduced by an effective force with $K_{\rm nm} \approx 250 - 270$ MeV [5, 6, 7, 8]. Twenty percent, of course, represents a rather large difference. The origin of this discrepancy is at present not understood.

It appears, however, that not all results of the relativistic models are in agreement either. By using the same type of non-linear RMF Lagrangians with scalar self-interactions as in Refs. [5, 6, 7, 8], in a series of recent papers [9, 10, 11] Piekarewicz has performed fully consistent relativistic RPA calculations of nuclear compressional modes. Basically the results can be summarized as follows. (1) The best description of both compressional modes – isoscalar monopole and dipole – is obtained by using a "soft" effective interaction with $K_{\rm nm} = 224$ MeV. (2) Effective interactions with compression moduli well above $K_{\rm nm} \approx 200$ MeV are not consistent with experimental data. (3) The compression modulus determined from the empirical excitation energies of the GMR depends on the nuclear matter symmetry energy, i.e. models with a lower symmetry energy at saturation density reproduce the GMR in ^{208}Pb by using a lower value of $K_{\rm nm}$. (4) The variance between the values of $K_{\rm nm}$ determined from non-relativistic and relativistic mean-field plus RPA calculations of GMR excitation energies can be attributed in part to the differences in the nuclear matter symmetry energy predicted by non-relativistic and relativistic models. In particular, in Ref. [11] it has been pointed out that, when the symmetry energy of the RMF models is softened to simulate the symmetry energy of Skyrme interactions, a lower value of $K_{\rm nm}$, consistent with the ones used in non-relativistic models, is required to reproduce the energy of the

GMR in ^{208}Pb.

Our own results strongly disagree with most of the conclusions of Refs. [9, 10, 11]. In addition to the results obtained in the RMF+RPA framework based on non-linear Lagrangians with scalar self-interactions [5, 6, 7, 8], in Ref. [12] we have carried out calculations of the isoscalar monopole, isovector dipole and isoscalar quadrupole response of ^{208}Pb, in the fully self-consistent relativistic RPA framework based on effective interactions with a phenomenological density dependence adjusted to nuclear matter and ground-state properties of spherical nuclei. The analysis of the isoscalar monopole response with density dependent coupling constants has shown that only interactions with the nuclear matter compression modulus in the range $K_{\rm nm} \approx 260 - 270$ MeV, reproduce the experimental excitation energy of the GMR in ^{208}Pb. In addition, the comparison with the experimental excitation energy of the isovector dipole resonance has constrained the volume asymmetry to the interval 34 MeV $\leq a_4 \leq$ 36 MeV.

Even though we cannot explain the difference between the nuclear matter compressibility predicted by non-relativistic and relativistic mean-field plus RPA calculations, at least we can clarify the apparent inconsistency of the results obtained in the relativistic framework. The purpose of this work is twofold. By using the RMF+RPA based on effective interactions with density-dependent meson-nucleon couplings, we will show: (1) contrary to the claims made in Ref. [11], our knowledge of the symmetry energy at low density is not that poor, and the volume asymmetry cannot be lowered to the range of values $a_4 \leq 30$ MeV, for which relativistic models with $K_{\rm nm} \leq 230$ MeV would reproduce the excitation energy of the GMR in ^{208}Pb and, (2) relativistic mean-field effective interactions adjusted to reproduce ground-state properties of spherical nuclei (binding energies, charge radii, differences between neutron and proton radii), cannot reproduce the excitation energies of GMR if $K_{\rm nm} \leq 250$ MeV. Therefore, we will reinforce our result that $K_{\rm nm} = 250$ MeV is the lower bound for the nuclear matter compression modulus in nuclear structure models based on the relativistic mean-field approximation.

II. EFFECTIVE INTERACTIONS WITH DENSITY-DEPENDENT MESON-NUCLEON COUPLINGS AND THE RELATIVISTIC RPA

The relativistic random phase approximation (RRPA) will be used to calculate the isoscalar monopole and isovector dipole strength distributions in ^{208}Pb. The RRPA represents the small amplitude limit of the time-dependent relativistic mean-field theory. A self-consistent calculations ensures that the same correlations which define the ground-state properties, also determine the behavior of small deviations from the equilibrium. The same effective Lagrangian generates the Dirac-Hartree single-particle spectrum and the residual particle-hole interaction. In Ref. [13] it has been shown that an RRPA calculation, consistent with the mean-field model in the $no-sea$ approximation, necessitates configuration spaces that include both particle-hole pairs and pairs formed from occupied states and negative-energy states. The contributions from configurations built from occupied positive-energy states and negative-energy states are essential for current conservation and the decoupling of the spurious state. In addition, configurations which include negative-energy states give an important contribution to the collectivity of excited states. In two recent studies [8, 14] we have shown that a fully consistent inclusion of the Dirac sea of negative energy states in the RRPA is crucial for a quantitative comparison with the experimental excitation energies of isoscalar giant resonances.

The second requisite for a successful application of the RRPA in the description of dynamical properties of nuclei is the use of effective Lagrangians with non-linear meson self-interactions, or Lagrangians characterized by density-dependent meson-nucleon vertex functions. Even though several RRPA implementations have been available for almost twenty years, techniques which enable the inclusion of non-linear meson interaction terms in the RRPA have been developed only recently [7, 9, 15]. In Ref. [12] the RRPA matrix equations have been derived for an effective Lagrangian with density-dependent meson-nucleon couplings.

Already in Ref. [5] we used Lagrangians with non-linear meson self-interactions in time-dependent RMF calculations of monopole oscillations of spherical nuclei. The energies of the GMR were determined from the Fourier power spectra of the time-dependent isoscalar monopole moments $< r^2 > (t)$. It has been shown that the GMR in heavy nuclei, as well as the empirical excitation energy curve $E_x \approx 80\ A^{-1/3}$ MeV, are best reproduced by an effective force with $K_{\rm nm} \approx 250 - 270$ MeV. This result has been confirmed by the RRPA calculation of Ref. [8]. In particular, the best results have been obtained with the NL3 effective interaction [16] ($K_{\rm nm} = 272$ MeV). Many calculations of ground-state properties and excited states, performed by different groups, have shown that NL3 is the best non-linear relativistic effective interaction so far, both for nuclei at and away from the line of β-stability. On the other hand, by using the same type of effective Lagrangians with non-linear isoscalar-scalar interaction terms, in the recent RRPA analysis of nuclear compressional modes of Refs. [9, 10] it has been suggested that models of nuclear structure having $K_{\rm nm}$ well above ≈ 200 MeV are likely to be in conflict with experiment.

Even though models with isoscalar-scalar meson self-interactions have been used in most applications of RMF to nuclear structure in the last fifteen years, they present well known limitations. In the isovector channel, for instance, these interactions are characterized by large values of the symmetry energy at saturation – 37.9 MeV for NL3 – as compared with the empirical value $a_4 = 30 \pm 4$ MeV. This is because the isovector channel of these effective forces is parameterized by a single constant: the ρ-meson nucleon coupling g_ρ. With a single parameter in the isovector channel, it is not possible to reproduce simultaneously the empirical value of a_4 and the masses of $N \neq Z$ nuclei. On the other hand, extensions of the RMF model that include additional interaction terms in the isoscalar and/or the isovector channels were not very successful, at least in nuclear structure applications. The reason is that the empirical data set of bulk and single-particle properties of finite nuclei can only constrain six or seven parameters in the general expansion of an effective Lagrangian [17]. Adding more interaction terms does not improve the description of finite nuclei. Rather, their coupling parameters and even their forms cannot be accurately determined.

Models based on an effective hadron field theory with medium dependent meson-nucleon vertices [18, 19, 20] present a very successful alternative to the use of nonlinear self-interactions. Such an approach retains the basic structure of the relativistic mean-field framework, but could be more directly related to the underlying microscopic description of nuclear interactions. In Ref. [21] we have extended the relativistic Hartree-Bogoliubov (RHB) model [22] to include density dependent meson-nucleon couplings. The effective Lagrangian is characterized by a phenomenological density dependence of the σ, ω and ρ meson-nucleon vertex functions, adjusted to properties of nuclear matter and finite nuclei. It has been shown that, in comparison with standard RMF effective interactions with non-linear meson-exchange terms, the density-dependent meson-nucleon couplings significantly improve the description of symmetric and asymmetric nuclear matter, and of ground-state properties of $N \neq Z$ nuclei.

The RRPA with density-dependent meson-nucleon couplings has been derived in Ref. [12]. Just as in the static case the single-nucleon Dirac equation includes the additional rearrangement self-energies that result from the variation of the vertex functionals with respect to the nucleon field operators, the explicit density dependence of the meson-nucleon couplings introduces rearrangement terms in the residual two-body interaction. Their contribution is essential for a quantitative description of excited states. By constructing families of interactions with some given characteristic (compressibility, symmetry energy, effective mass), it has been shown how the comparison of the RRPA results on multipole giant resonances with experimental data, can be used to constrain the parameters that characterize the isoscalar and isovector channel of the density-dependent effective interactions. In particular, the GMR in ^{208}Pb requires the compression modulus to be in the range $K_{\rm nm} \approx 260 - 270$ MeV, and the isovector GDR in ^{208}Pb is only reproduced with the volume asymmetry in the interval 34 MeV $\leq a_4 \leq$ 36 MeV. We have not, however, analyzed the influence of the symmetry energy on the range of allowed values of $K_{\rm nm}$. We have also not tried to correlate the RRPA results for the isovector GDR with data on neutron radii, although it has been shown in Ref. [21] that RHB calculations with the density-dependent effective interaction DD-ME1 ($a_4 = 33.1$ MeV) reproduce the available data on differences between neutron and proton radii for ^{208}Pb and several Sn isotopes.

III. NUCLEAR MATTER COMPRESSIBILITY AND SYMMETRY ENERGY

By performing fully consistent RMF plus RRPA calculations of nuclear ground-state properties and excitation energies of giant resonances, in this section we will try to correlate the nuclear matter symmetry energy and the nuclear matter compression modulus of relativistic mean-field effective interactions.

In Ref. [11] Piekarewicz has used the standard RMF effective interactions with isoscalar-scalar meson self-interactions, and with compression moduli in the range $K_{\rm nm} \approx 200 - 300$ MeV, to compute the distribution of isoscalar monopole strength in ^{208}Pb. The main result of his analysis is that, when the symmetry energy is artificially softened, in an attempt to simulate the symmetry energy of Skyrme interactions, a lower value for the compression modulus is obtained, consistent with the predictions of non-relativistic Hartree-Fock plus RPA calculations.

As we have already emphasized in the previous section, the RMF models with isoscalar-scalar meson self-interactions are characterized by large values of the symmetry energy at saturation volume asymmetry. When adjusting such an effective interaction to properties of nuclear matter and bulk ground-state properties of finite nuclei (binding energy, charge radius) it would be clearly desirable to bring down the value of a_4 to its empirical value $a_4 = 30 \pm 4$ MeV. This, however, is not feasible. The simple reason is that, if a_4 is brought below $\approx 36-37$ MeV by simply reducing the single coupling constant g_ρ in the isovector channel, then it is no longer possible to reproduce the relative positions of the neutron and proton Fermi levels in finite nuclei, i.e. the calculated masses of $N \neq Z$ nuclei display large deviations from the experimental values. The binding energies are only reproduced if a density dependence is included in the ρ-meson coupling, or a non-linear ρ-meson self-interaction is included in the model. If the interaction, however, is adjusted to a single nucleus, as it was done in Ref. [11] for ^{208}Pb, then even the standard RMF interactions contain enough parameters to obtain almost any combination of symmetry energy and nuclear matter compressibility.

In the present analysis we have used effective interactions with density-dependent meson-nucleon vertex functions. For the density dependence of the meson-nucleon couplings we adopt the functionals used in Refs.[19, 20, 21]. The coupling of the σ-meson and ω-meson to the nucleon field reads

$$g_i(\rho) = g_i(\rho_{\rm sat}) f_i(x) \quad \text{for} \quad i = \sigma, \omega \; , \tag{2}$$

where

$$f_i(x) = a_i \frac{1 + b_i (x + d_i)^2}{1 + c_i (x + d_i)^2} \tag{3}$$

is a function of $x = \rho/\rho_{\rm sat}$, and $\rho_{\rm sat}$ denotes the baryon density at saturation in symmetric nuclear matter. The eight real parameters in (3) are not independent. The five constraints $f_i(1) = 1$, $f''_\sigma(1) = f''_\omega(1)$, and $f''_i(0) = 0$, reduce the number of independent parameters to three. Three additional parameters in the isoscalar channel are: $g_\sigma(\rho_{\rm sat})$, $g_\omega(\rho_{\rm sat})$, and m_σ - the mass of the phenomenological sigma-meson. For the ρ-meson coupling the functional form of the density dependence is suggested by Dirac-Brueckner calculations of asymmetric nuclear matter [23]

$$g_\rho(\rho) = g_\rho(\rho_{\rm sat}) \exp\left[-a_\rho(x-1)\right] \ . \tag{4}$$

The isovector channel is parameterized by $g_\rho(\rho_{\rm sat})$ and a_ρ. Usually the free values are used for the masses of the ω and ρ mesons: $m_\omega = 783$ MeV and $m_\rho = 763$ MeV. In principle one could also consider the density dependence of the meson masses. However, since the effective meson-nucleon coupling in nuclear matter is determined by the ratio g/m, the choice of a phenomenological density dependence of the couplings makes an explicit density dependence of the masses redundant.

Obviously, the framework of density-dependent interactions is more general than the standard RMF models, and it encloses models with non-linear meson self-interactions in the isoscalar-scalar, isoscalar-vector, and isovector-vector channels. The eight independent parameters, seven coupling parameters and the mass of the σ-meson, are adjusted to reproduce the properties of symmetric and asymmetric nuclear matter, binding energies and charge radii of spherical nuclei. For the open-shell nuclei pairing correlations are treated in the BCS approximation with empirical pairing gaps (five-point formula).

In order to investigate possible correlations between the nuclear matter symmetry energy and the compression modulus, we have constructed three families of interactions, with $K_{\rm nm} =$ 230, 250, and 270 MeV, respectively. For each value of $K_{\rm nm}$ we have adjusted five interactions with $a_4 = 30$, 32, 34, 36 and 38 MeV, respectively. These interactions have been fitted to properties of nuclear matter (the binding energy $E/A = 16$ MeV (5%), the saturation density $\rho_{\rm sat} = 0.153$ fm^{-3} (5%), the compression modulus $K_{\rm nm}$ (0.1%), and the volume asymmetry a_4 (0.1%)), and to the binding energies (0.1%) and charge radii (0.2%) of ten spherical nuclei: ^{16}O, ^{40}Ca, ^{90}Zr, ^{112,116,124,132}Sn, and ^{204,208,214}Pb, as well as to the differences between neutron and proton radii (10%) for the nuclei ^{116}Sn, ^{124}Sn and ^{208}Pb. The values in parentheses correspond to the error bars used in the fitting procedure. Note that, in order to fix an interaction with particular values of $K_{\rm nm}$ and a_4, a very small error bar of only 0.1% is used for these two quantities. For each family of interactions with a given $K_{\rm nm}$ and for each a_4, in Fig. 1 we plot the differences, expressed as a percentage, between the calculated and experimental binding energies [24]. The corresponding deviations of charge radii are shown in Fig. 2. The results are rather good. Most deviations of the binding energies are $\leq 0.2\%$, and the differences between the calculated and experimental charge radii [25] are $\leq 0.3\%$, except for ^{40}Ca. It should be emphasized that, by using the standard RMF models with only isoscalar-scalar meson self-interactions, it is simply not possible to construct a set of interactions with this span of values of $K_{\rm nm}$ and a_4 and the same quality of deviations of masses and charge radii from experimental values.

For the three sets of interactions with $K_{\rm nm}$ = 230, 250, and 270 MeV, in Fig. 3 we display the corresponding nuclear matter symmetry energy curves for each choice of the volume asymmetry a_4. The symmetry energy can be parameterized

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_{\rm sat}^2}\,(\rho - \rho_{\rm sat}) + \frac{\Delta K_0}{18\rho_{\rm sat}^2}\,(\rho - \rho_{\rm sat})^2 + \cdots \tag{5}$$

The parameter p_0 defines the linear density dependence of the symmetry energy, and ΔK_0 is the correction to the incompressibility.

In finite nuclei, among other quantities, the symmetry energy directly determines the differences between the neutron and the proton radii. In a recent study of neutron radii in non-relativistic and covariant mean-field models [26], the linear correlation between the neutron skin and the symmetry energy has been analyzed. In particular, the analysis has shown that there is a very strong linear correlation between the neutron skin thickness in ^{208}Pb and the individual parameters that determine the symmetry energy $S_2(\rho)$: a_4, p_0 and ΔK_0. The empirical value of $r_n - r_p$ in ^{208}Pb (0.20 ± 0.04 fm from proton scattering data [27], and 0.19 ± 0.09 fm from the alpha scattering excitation of the isovector giant dipole resonance [28]) places the following constraints on the values of the parameters of the symmetry energy: $a_4 \approx 30 - 34$ MeV, 2 Mev/fm^3 $\leq p_0 \leq$ 4 Mev/fm^3, and -200 MeV $\leq \Delta K_0 \leq -50$ MeV.

For the three sets of interactions with $K_{\rm nm}$ = 230, 250, and 270 MeV, in the two lower panels of Fig. 4 we plot the coefficients p_0 and ΔK_0 as functions of the volume asymmetry a_4. As we have shown in Ref. [12], in order to reproduce the bulk properties of spherical nuclei, larger values of a_4 necessitate an increase of p_0. It is important to note that only in the interval 32 MeV $\leq a_4 \leq$ 36 MeV, both p_0 and ΔK_0 are found within the bounds determined by the value of $r_n - r_p$ in ^{208}Pb. The increase of p_0 with a_4 implies a transition from a parabolic to an almost linear density dependence of S_2 in the density region $\rho \leq 0.2$ fm^{-3} (see Fig. 3). This means, in particular, that the increase of the asymmetry energy at saturation point will produce an effective decrease of S_2 below $\rho \approx 0.1$ fm^{-3}. In Refs. [12, 29] it has been shown that, as a result of the increase of p_0 with a_4, the excitation energy of the isovector GDR decreases with increasing $S_2(\rho_{\rm sat}) \equiv a_4$, because this increase implies a decrease of S_2 at low densities characteristic for surface modes. This effect is illustrated in the upper panel of Fig. 4, where we plot the calculated excitation energies of the isovector GDR in ^{208}Pb as functions of a_4, for each set of interactions with $K_{\rm nm} = 230$, 250, and 270 MeV. The calculated centroids of the Lorentzian folded strength distributions are shown in comparison with the experimental value of 13.3 ± 0.1 [30]. The RRPA excitation energies of the isovector GDR decrease linearly with a_4, and the experimental value favors, for all three families of interactions, the interval 34 MeV $\leq a_4 \leq$ 36 MeV for the volume asymmetry.

The results of fully consistent RRPA calculations of the isoscalar monopole response in ^{208}Pb are shown in the upper panel of Fig. 5, where we plot the excitation energies of the GMR for the three families of interactions with $K_{\rm nm} = 230$, 250, and 270 MeV, respectively, as functions of the volume asymmetry a_4. The shaded area denotes the experimental value: $E = 14.17 \pm 0.28$ MeV [4]. For each interaction, in the lower panel we plot the corresponding result for the difference of the neutron and proton radius in ^{208}Pb, in comparison with, at present, the best experimental value: 0.20 ± 0.04 fm from proton scattering data [27]. The calculated radii practically do not depend on the compressibility but, of course, display a strong linear dependence on the volume asymmetry a_4. The comparison with the experimental estimate limits the possible values of the symmetry energy at saturation to 32 MeV $\leq a_4 \leq 36$. This interval for a_4 is somewhat wider than the range deduced from the isovector GDR but, nevertheless, both quantities exclude values $a_4 \leq 30$ MeV and $a_4 \geq 38$ MeV. Coming back to the GMR (upper panel of Fig. 5), we notice that only the set of interactions with $K_{\rm nm} = 270$ MeV reproduces the experimental excitation energy of the GMR for all values of the volume asymmetry a_4. With $K_{\rm nm} = 250$ MeV, only for the two lowest values of a_4 the RRPA results for the GMR excitation energy are found within the bounds of the experimental value. $K_{\rm nm} = 250$ MeV is obviously the lower limit, if not even too low, for the nuclear matter compression modulus of the relativistic mean-field effective interactions. This result is completely in accordance with our previous results obtained with relativistic effective forces with non-linear meson self-interactions [5, 8], and with density-dependent interactions [12]. The calculation absolutely excludes the set of interactions with $K_{\rm nm} = 230$ MeV, for any value of a_4. The calculated energies are simply too low compared with the experimental value.

Finally, in Fig. 6 we plot, for all three families of interactions, the calculated differences between neutron and proton radii of Sn isotopes, as functions of the mass number, in comparison with available experimental data [31]. Similar to the result obtained for ^{208}Pb, the calculated values practically do not depend on the nuclear matter compressibility. While all the interactions reproduce the isotopic trend of the experimental data, and we also notice that the error bars are rather large, nevertheless the comparison excludes values $a_4 \leq 30$ MeV and $a_4 \geq 38$ MeV.

IV. SUMMARY AND CONCLUSIONS

In this work we have clarified the apparent inconsistency of the values of the nuclear matter compression modulus deduced from relativistic RPA calculations of the giant monopole resonance (GMR) in ^{208}Pb. By using standard RMF effective Lagrangians with non-linear isoscalar scalar meson self-interactions, in a number of studies (time-dependent RMF, relativistic RPA) we have shown that the experimental data on GMR in heavy nuclei, as well as the empirical excitation energy curve $E_x \approx 80\ A^{-1/3}$ MeV, are best reproduced by an effective force with $K_{\rm nm} \approx 250 - 270$ MeV [5, 6, 7, 8]. The best results have been obtained with the well known effective interaction NL3 [16] ($K_{\rm nm} = 272$ MeV). On the other hand, by using the same type of effective Lagrangians in the fully consistent relativistic RPA calculations of the isoscalar monopole response, Piekarewicz has concluded that RMF models with $K_{\rm nm}$ well above ≈ 200 MeV will be in conflict with experiment [9, 10], and that the discrepancy between the nuclear matter compressibility determined from non-relativistic and relativistic mean-field plus RPA calculations of GMR excitation energies can be attributed in part to the differences in the nuclear matter symmetry energy predicted by non-relativistic and relativistic models [11].

It is well known, however, that the standard RMF Lagrangians, with meson self-interactions only in the isoscalar channel, are characterized by large values of the symmetry energy at saturation (volume asymmetry) a_4. In fact, if the effective interaction in the isovector channel is parameterized by the single ρ-meson nucleon coupling constant, it is not possible to simultaneously reduce the value of a_4 below $\approx 36 - 37$ MeV, and still reproduce the experimental binding energies of $N \neq Z$ nuclei. In this framework it is simply not possible to construct effective interactions with the volume asymmetry in the range of empirical values $a_4 = 30 \pm 4$ MeV.

In the present analysis we have used effective interactions with density-dependent meson-nucleon couplings. RMF models based on an effective hadron field theory with medium dependent meson-nucleon vertices [18], provide a much better description of symmetric and asymmetric nuclear matter, and of ground-state properties of $N \neq Z$ nuclei [19, 20, 21]. In order to investigate possible correlations between the volume asymmetry and the nuclear matter compression modulus, we have constructed three sets of effective interactions with $K_{\rm nm} = 230$, 250, and 270 MeV, and for each value of $K_{\rm nm}$ we have adjusted five interactions with $a_4 = 30$, 32, 34, 36 and 38 MeV, respectively. The interactions have been fitted to reproduce the nuclear matter saturation properties, as well as the ground states of ten spherical nuclei.

By employing the fully consistent RMF plus RRPA framework with density-dependent effective interactions, we have computed the isoscalar monopole and the isovector dipole response of ^{208}Pb, as well as the differences between the neutron and proton radii for ^{208}Pb and several Sn isotopes. The comparison of the calculated excitation energies with the experimental data on the GMR and isovector GDR in ^{208}Pb has shown that: (i) only for $K_{\rm nm} = 270$ MeV the RRPA calculation reproduces the experimental excitation energy of the GMR for all values of the volume asymmetry a_4, (ii) $K_{\rm nm} = 250$ MeV represents the lower limit for the nuclear matter compression modulus of the relativistic mean-field effective interactions, (iii) the isovector GDR constrains the volume asymmetry to the interval 34 MeV $\leq a_4 \leq$ 36 MeV. In comparison with the available experimental data, the calculated differences between neutron and proton radii indicate that the volume asymmetry should be in the range 32 MeV $\leq a_4 \leq$ 36 MeV, and reinforce our conclusion that a_4 cannot be lowered to a range of values for which relativistic models with $K_{\rm nm} \leq 230$ MeV would reproduce the excitation energy of the GMR in ^{208}Pb. The disagreement between the nuclear matter compression moduli predicted by non-relativistic and relativistic mean-field plus RPA calculations, cannot be explained by the differences in the volume asymmetry of the non-relativistic and relativistic mean-field models.

The present analysis has confirmed our earlier results that the nuclear matter compression modulus of structure models based on the relativistic mean-field approximation should be restricted to the interval $K_{\rm nm} \approx 250 - 270$ MeV, or even slightly higher.

In addition, we have also shown that, for the relativistic mean-field models, the isovector GDR and the available data on differences between neutron and proton radii limit the range of the nuclear matter symmetry energy at saturation to 32 MeV $\leq a_4 \leq$ 36 MeV. It appears that the GDR favors the high end of this interval, but we stress the fact that in the present analysis we have not taken into account the influence of the effective mass on the calculated excitation energy of the GDR. This is, however, an effect which really goes beyond the mean-field approximation. Rather, more accurate data on neutron radii in heavy nuclei would provide very useful information of the isovector channel of the effective RMF interactions. On the other hand, as it has been shown in Ref. [1], there is no correlation between the effective mass and the excitation energy of the GMR. The choice of the effective mass, therefore, cannot influence the nuclear matter compressibility extracted from the GMR.

ACKNOWLEDGMENTS

This work has been supported in part by the Bundesministerium für Bildung und Forschung under the project 06 TM 979 and by the Deutsche Forschungsgemeinschaft.

---

[1] J.P. Blaizot, J.F. Berger, J. Dechargé, and M. Girod, Nucl. Phys. **A591**, 435 (1995).

[2] J.P. Blaizot, Phys. Rep. **64**, 171 (1980).

[3] M. Farine, J.M. Pearson, and F. Tondeur, Nucl. Phys. **A615**, 135 (1997).

[4] D.H. Youngblood, H.L. Clark, and Y.-W. Lui, Phys. Rev. Lett. **82**, 691 (1999).

[5] D. Vretenar, G.A. Lalazissis, R. Behnsch, W. Pöschl, and P. Ring, Nucl. Phys. **A621**, 853 (1997).

[6] D. Vretenar, P. Ring, G.A. Lalazissis, and N. Paar, Nucl. Phys. **A649**, 29c (1999).

[7] D. Vretenar, A. Wandelt, and P. Ring, Phys. Lett. **B487**, 334 (2000).

[8] Z.Y. Ma, N. Van Giai, A. Wandelt, D. Vretenar, and P. Ring, Nucl. Phys. **A686**, 173 (2001).

[9] J. Piekarewicz, Phys. Rev. C **62**, 051304(R) (2000).

[10] J. Piekarewicz, Phys. Rev. C **64**, 024307 (2001).

[11] J. Piekarewicz, Phys. Rev. C **66**, 034305 (2002).

[12] T. Nikšić, D. Vretenar, and P. Ring, Phys. Rev. C **66**, 064302 (2002).

[13] J.F. Dawson and R.J. Furnstahl, Phys. Rev. C **42**, 2009 (1990).

[14] P. Ring, Zhong-yu Ma, Nguyen Van Giai, D. Vretenar, A. Wandelt, and Li-gang Cao, Nucl. Phys. **A694**, 249 (2001).

[15] Z.Y. Ma, N. Van Giai, H. Toki, and M. L'Huillier, Phys.Rev. C **42**, 2385 (1997).

[16] G.A. Lalazissis, J. König, and P. Ring, Phys. Rev. C **55**, 540 (1997).

[17] R.J. Furnstahl and B.D. Serot, Nucl. Phys. A **671**, 447 (2000).

[18] C. Fuchs, H. Lenske, and H.H. Wolter, Phys. Rev. C **52**, 3043 (1995).

[19] S. Typel and H.H. Wolter, Nucl. Phys. **A656**, 331 (1999).

[20] F. Hofmann, C.M. Keil, and H. Lenske, Phys. Rev. C **64**, 034314 (2001).

[21] T. Nikšić, D. Vretenar, P. Finelli, and P. Ring, Phys. Rev. C **66**, 024306 (2002).

[22] W. Pöschl, D. Vretenar, G.A. Lalazissis, and P. Ring, Phys. Rev. Lett. **79**, 3841 (1997).

[23] F. de Jong and H. Lenske, Phys. Rev. C **57**, 3099 (1998).

[24] G. Audi and A. H. Wapstra, Nucl. Phys. **A595**, 409 (1995).

[25] E.G. Nadiakov, K.P. Marinova, Yu.P. Gangrsky, At. Data Nucl. Data Tables **56**, 133 (1994).

[26] R.J. Furnstahl, Nucl. Phys. **A706**, 85 (2002).

[27] V.E. Starodubsky and N.M. Hintz, Phys. Rev. C **49**, 2118 (1994).

[28] A. Krasznahorkay et al., Nucl. Phys. **A567**, 521 (1994).

[29] P.-G. Reinhard, Nucl. Phys. **A649**, 305c (1999).

[30] J. Ritman et al., Phys. Rev. Lett. **70**, 533 (1993).

[31] A. Krasznahorkay et al., Phys. Rev. Lett. **82**, 3216 (1999).

FIG. 1: The deviations (in percent) of the theoretical binding energies of ten spherical nuclei, calculated with the three families of interactions with $K_{\rm nm} = 230$, 250, and 270 MeV, from the empirical values [24]. The legend relates the different symbols to the volume asymmetries of the corresponding effective interactions.

FIG. 2: Same as in Fig. 1, but for the charge radii compared with the experimental values [25].

FIG. 4: The isovector GDR excitation energy of ^{208}Pb (upper left panel),the parameter p_0 of the linear density dependence of the nuclear matter asymmetry energy (lower left), and the correction to the incompressibility ΔK_0 (lower right), as functions of the volume asymmetry a_4. The shaded area denotes the experimental isovector GD resonance energy 13.3 ± 0.1 MeV. The three sets of symbols correspond to the families of interactions with $K_{\rm nm} = 230$, 250, and 270 MeV.

FIG. 5: The RRPA excitation energies of the GMR in ^{208}Pb as functions of the volume asymmetry a_4, calculated for the three sets of interactions with $K_{\rm nm} = 230$, 250, and 270 MeV. The theoretical centroids are shown in comparison with the experimental excitation energy of the monopole resonance: $E = 14.17 \pm 0.28$ MeV [4]. In the lower panel the corresponding results for the difference of the neutron and proton radius in ^{208}Pb, are plotted in comparison with the experimental value 0.20 ± 0.04 fm [27].

FIG. 6: The calculated differences between neutron and proton radii of Sn isotopes, calculated with the three sets of interactions with $K_{\rm nm} = 230$, 250, and 270 MeV, and with the values of the volume asymmetry $a_4 = 30$, 32, 34, 36 and 38 MeV. The theoretical values are compared with experimental data from Ref. [31].

FIG. 3: $S_2(\rho)$ coefficient (5) of the quadratic term of the energy per particle of asymmetric nuclear matter, calculated with the three families of effective interactions with $K_{\rm nm} = 230$, 250, and 270 MeV. The legend relates the different curves to the volume asymmetries of the corresponding effective interactions.